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Invited Paper

Swing principle for deployment of a tether-assisted return mission of a re-entry capsule

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ABSTRACT

Dynamics and control of a tether-assisted return mission of a re-entry capsule are considered. Efficiency of the braking process of the capsule depends on a deflection angle of the tether from a local vertical before separating the capsule from the tether. The aim is of this paper to find a tether length control law that allows to increase the deflection angle of the tether from the local vertical. The control law is based on a principle of a swing. This control law may be applied to the final phase to two possible options to perform a tether-assisted deorbit maneuver: static and dynamic release. An approximate analytical solution for the deflection angle from the local vertical is obtained for the control law. The numerical simulations have shown that the application of the control law allows a reduction in the required tether length of a tether-assisted deorbit maneuver. The proposed control law can be applied to develop new space tethered systems.

1. Introduction

Space tethers have received more attention in recent decades, with many articles and books [1–3] available. The fundamental paper by Beletsky and Levin [1] played an important role in providing the basis for the study of tethered system dynamics. Tethered systems offer numerous benefits to modern spacecrafts. One central advantage is that they use less fuel. Another advantage is that tethers allow payload delivery from the Earth's orbit [1–7]. There are two essentially different approaches to the tether release with the re-entry capsule: static and dynamic deployment [4]. Static deployment is the slow release of the tether close to the local vertical. Dynamic deployment means that the decrease of payload velocity comes from the swinging of the tether impacted by the Coriolis force acting on it [4]. Successful experiments of payload delivery occurred in 1993 and 2007 [5–7]. The 1993 mission, SEDS-1, used static deployment, while the 2007 mission, YES2, used dynamic deployment. The YES2 mission was demonstrated an ability to return a re-entry capsule to the Earth using a tether. Using a swinging tether releasing the re-entry capsule from an end of vertical tether 30 km below mother satellite orbit provided braking the re-entry capsule.

The aim of this paper is to develop a control law for the final phase of the deployment of the tether system for payload delivery to Earth's surface. This leads to an increase of a deflection angle of a tether from a local vertical and hence reduces perigee altitude of a re-entry trajectory of a capsule [7]. This control law is based on the principle of a swing and the control law should be applicable in cases where the initial deployment was performed in static or dynamic modes. In both cases the tether is required to reach the desired value of a deflection angle of the tether from the local vertical.

Consider the essence of the proposed control law. Suppose that after the initial deployment the tether is at the point C as shown in Fig. 1. In other words, the tether

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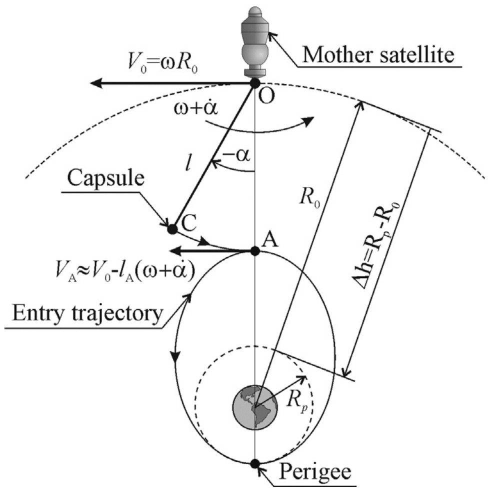

Fig. 1. Swinging release of a capsule from a tether.

has reached the leftmost position

$$\alpha<0, \dot\alpha=0. \tag{1}$$

The amplitude of the deflection angle α_m can be increased, if we use the following control law

$$\dot l_0=-\lambda\dot\alpha, \tag{2}$$

where l_0 is total length of the tether and $\lambda>0$ is a constant coefficient. A similar control law has been used in the tasks of gravitational stability of a satellite [8] and of a mathematical pendulum [9].

The physical nature of this phenomenon is as follows: Suppose the attached capsule is at point C (Fig. 1) and the conditions (1) are satisfied. According to the law (2) during the reverse motion of the capsule from point C the tether retracts $\dot l<0$. This gives rise to the Coriolis force $\Phi_C=2m_c\dot\alpha\dot l$ (m_c is mass of the capsule) which increases the speed of the capsule toward the local vertical. During the reverse motion of the capsule from the right end position, the tether begins to release $\dot l>0$. In this case, the Coriolis force increases the capsule velocity only in the opposite direction. Because this phenomenon increases the tether oscillation amplitude, the capsule should separate from the tether at point A, when

$$\alpha=0, \dot\alpha>0. \tag{3}$$

We note that the tether should be stretched throughout the motion.

2. Tethered system model

Consider two-dimensional motion of the tethered system in the orbital plane. The tethered system consists of a mother satellite, the capsule, and a viscoelastic tether between the two (Fig. 1). The mother satellite and the capsule are modeled as material points which have masses m_m and m_c respectively.

We introduce the following assumptions:

1. The mass of the capsule significantly less than the mass of the mother satellite

$$m_c\ll m_m. \tag{4}$$

2. The tether is weightless

$$m_t=0. \tag{5}$$

3. The tether length l is much smaller than the mother satellite orbital radius

$$l\ll R_0=\frac{p}{1+e\cos\theta}, \tag{6}$$

where e is orbital eccentricity, p is orbital parameter and θ is true anomaly.

Tether tension force is expressed as

$$T=\frac{EA}{l_0}(l-l_0)+\frac{C}{l_0}\dot l, \tag{7}$$

where EA is a stiffness of the tether, C is a damping constant.

Taking into account the assumptions (4–6) equations of the motion of the capsule relative to the mother satellite can be written as

$$\ddot\alpha+\ddot\theta+2\frac{\dot l}{l}(\dot\alpha+\omega)+3\frac{\mu}{R_0^3}\sin\alpha\cos\alpha=0, \tag{8}$$

$$m_c\ddot l=2m_c\omega^2 l\cos^2\alpha+m_c\dot\alpha^2 l-T, \tag{9}$$

$$\dot l_0=-\lambda\dot\alpha. \tag{10}$$

Where $\omega=\sqrt{\mu R_0^{-3}}$, μ is the gravitational constant of the Earth.

For the convenience of analysis, the independent variable can be changed from time t to true anomaly θ [10,11]. Then Eqs. (8–10) can be re-written as

$$\alpha''+\frac{3}{1+e\cos\theta}\sin\alpha\cos\alpha+2\left(\frac{l'}{l}-\frac{2e\sin\theta}{1+e\cos\theta}\right)(\alpha'+1)=0, \tag{11}$$

$$l''-\frac{2e\sin\theta}{1+e\cos\theta}l'-l\alpha'^2-\frac{1}{N^2(1+e\cos\theta)^4}\left[\frac{2g_0 l}{R_0}\cos^2\alpha-\frac{EA}{m_c l_0}(l-l_0)\right]+\frac{C}{m_c l_0 N(1+e\cos\theta)^2}l'=0, \tag{12}$$

$$l_0'=-\lambda\alpha', \tag{13}$$

where $()'=d()/d\theta$ is derivative with respect to true anomaly, $N=n/(1+e^2)^{3/2}$, n is the mother satellite's average orbital angular velocity, g_0 is gravitational acceleration of the mother satellite (Fig. 1). Note that Eq. (11) coincides with the corresponding Eq. given by [10].

If the cable is considered inextensible then Eqs. (11–13) reduced to one equation

$$\alpha''+\frac{3}{1+e\cos\theta}\sin\alpha\cos\alpha-2\left(\frac{\lambda\alpha'}{l_0}+\frac{2e\sin\theta}{1+e\cos\theta}\right)(\alpha'+1)=0. \tag{14}$$

In this case, the tether tension force is

$$T = m_c\left[N^2(1+e\cos\theta)^4\left(\lambda\alpha''+\frac{2e\lambda\alpha'}{1+e\cos\theta}\sin\theta+\alpha'^2 l_0\right)+\frac{2g_0 l_0}{R_0}\cos^2\alpha\right]. \tag{15}$$

3. Averaged equation and analytical solution

To find an approximate analytical solution of Eq. (14) these additional assumptions should be introduced.

1. The mother satellite moves in a circular orbit

$$e=0, \dot{\theta}=\omega=\sqrt{\mu R_0^{-3}}=const. \tag{16}$$

2. The control coefficient λ is much smaller than the tether length

$$\varepsilon=\frac{\lambda}{l_0}\ll 1. \tag{17}$$

Taking into account the assumptions (16) and (17), we can rewrite Eq. (14) as

$$\alpha''+\nu^2\sin\alpha\cos\alpha=2\varepsilon(\alpha'+1)\alpha', \tag{18}$$

where $\nu^2=3$.

If we set $\varepsilon=0$ in Eq. (18) we obtain the unperturbed equation

$$\alpha''+\nu^2\sin\alpha\cos\alpha=0. \tag{19}$$

Now we write the energy integral for Eq. (19)

$$\frac{\alpha'^2}{2}-\frac{\nu^2}{4}\cos 2\alpha=W. \tag{20}$$

Taking into account the Eq. (18) the energy integral (20) may be differentiated

$$W'=2\varepsilon(\alpha'+1)\alpha'^2,$$

and averaging the right-hand side of this equation over the period of the variable θ

$$T_\theta=\oint d\theta, \tag{21}$$

we get

$$W'=\frac{2\varepsilon}{T_\theta}\oint(\alpha'+1)\alpha'^2 d\theta. \tag{22}$$

Solving Eq. (20) with respect to

$$(\alpha')=\pm\sqrt{2\left(W+\frac{\nu^2}{4}\cos 2\alpha\right)}.$$

Eqs. (22) and (21) can be written as

$$W'=\frac{8\varepsilon}{T_\theta}\int_0^{\alpha_m}\sqrt{2\left(W+\frac{\nu^2}{4}\cos 2\alpha\right)}d\alpha, \tag{23}$$

$$T_\theta=\oint d\theta=4\int_0^{\alpha_m}\frac{d\alpha}{\sqrt{2\left(W+\frac{\nu^2}{4}\cos 2\alpha\right)}}. \tag{24}$$

The integrals in the right-hand sides of these equations are elliptic integrals. The change of variable $\sin\alpha=k\sin\varphi$ ($k=\sin\alpha_m$)

converts these integrals to the complete elliptic integrals of the first and second kind [12]

$$K(k)=\int_0^{\frac{\pi}{2}}\frac{d\varphi}{\sqrt{\left(1+k^2\sin^2\varphi\right)}}, E(k)=\int_0^{\frac{\pi}{2}}\sqrt{\left(1+k^2\sin^2\varphi\right)}d\varphi.$$

This finally leads us to the following equations

$$W'=8\varepsilon\left[\frac{E(k)}{K(k)}-\left(1-k^2\right)\right], \tag{25}$$

$$T_\theta=\frac{4}{\nu}K(k).$$

From Eq. (20) we have

$$W(\alpha,\dot{\alpha})=W(\alpha_m,\dot{\alpha}=0)=2\sin^2\alpha_m-1=2k^2-1=2x-1, \tag{26}$$

where $x=k^2=\sin^2\alpha_m$ is the new variable.

The variable substitution (26) in Eq. (25) gives

$$x'=4\varepsilon\left[\frac{E(\sqrt{x})}{K(\sqrt{x})}-(1-x)\right]. \tag{27}$$

Eq. (27) is approximated by a cubic polynomial

$$\frac{dx}{d\theta}=-\frac{\varepsilon}{8}x(x^2+2x+16). \tag{28}$$

Separating the variables in Eq. (28) and integrating it, we get

$$4a\varepsilon(\theta-\theta_0)=\left[(\sqrt{a}-a)\ln(\sqrt{a}+1+x)-(\sqrt{a}+a)\ln(\sqrt{a}-1-x)+2\ln(x)\right]_{\sin^2\alpha_{m0}}^{\sin^2\alpha_m} \tag{29}$$

where $a=17$, $\alpha_{m0}=\alpha_m(\theta_0)$

This solution establishes a relationship between the amplitude of the tether oscillation α_m and true anomaly θ.

4. Numerical analyses

In order to check the effectiveness of the control law given by (2), several numerical techniques are used. The numerical results are based on the numerical integration of Eq. (14) using an explicit fourth-order Runge–Kutta method.

The change in altitude of the capsule if cut from the tether is given by [7]

$$\Delta h=R_p-R_0=\frac{(R_A V_A)^2}{2\mu-R_A V_A^2}-R_0. \tag{30}$$

where R_p is a perigee height of a re-entry trajectory, $R_A=R_0-l_A$, $V_A\approx\omega\left[R_0-l_A(\alpha_A'+1)\right]$, as shown in Fig. 1.

We choose YES2 mission [7] as an example for comparison with the proposed control law. It is known that for the YES2 mission the change in altitude was

$$\Delta h_{YES2}\approx -330\text{ km}, \tag{31}$$

when the amplitude $\alpha_m=40$ deg, the tether length $l=30$ kmand $R_0=6645$ km [7].

The coefficient control λ, so that the change in altitude was similar to YES2 mission. Table 1 presents parameters of the tether system. The tether tension force was calculated by Eq. (15).

The initial conditions are

$$\alpha_0 = -40\text{ deg}, \alpha'_0 = 0, l_0 = 25\text{ km}. \quad (32)$$

Table 2 shows the results of simulation for different values of the coefficient control λ. Based on the data from Table 2, we can make some conclusions:

1. The tether tension force does not exceed 2.04 N,
2. The tether length increases by not more than 0.4 km and retracts by not more than 2.04km, when $l_0 = 25$ km,
3. The maximum speed of release of the tether is less than 2.0 m/s.

Note that if the control coefficient $\lambda = 750$ m, then the tether length can be reduced to approximately 5 km as compared with YES2 mission.

Consider behavior of the tether system under the control law (2), taking into account the viscoelastic properties of the tether and orbital eccentricity. The tether properties are approximate and are taken to be: stiffness $EA = 6000$ N, damping constant $C = 4000$ N s. The perigee altitude is 249 km, and the apogee altitude is 285 km. Thus, the orbit semimajor axis is 6645 km and the orbital eccentricity is 0.0027 [7]. All other parameters are contained in Table 1 and the initial conditions are given by (32). Fig. 2 shows the results of numerical integration of Eqs. (11)–(13). The change in altitude equals $\Delta h = -329.2$ km for this numerical experiment. Fig. 2c depicts that the tether tension force is less than 2N and the tether remains stretched during the deployment process. As can be seen in Fig. 2 and from Table 2, that the small eccentricity $e = 0.0027$ and the viscoelastic properties of the tether does not lead to significant differences from the results of the numerical integration of Eq. (14).

To illustrate the ability of the control law to swing the tether from the nearly vertical position we take the following initial conditions

$$\alpha_0 = -1\text{deg}, \alpha'_0 = 0, l_0 = 25\text{ km}. \quad (33)$$

Table 1
Parameters of the tethered system.

Parameter	Value	Parameter	Value
Orbital radius R_0	6645 km	Mass of the mother satellite m_m	6530 kg
Eccentricity e	0	Mass of the capsule m_c	12 kg

Fig. 3 shows the closeness of the numerical solution obtained using the averaged Eq. (27) and the analytical solution (29) to the numerical solution obtained using the original Eq. (18).

the original Eq. (18), and the averaged Eq. (27) and analytical solution (29) for $\lambda = 750$ m.

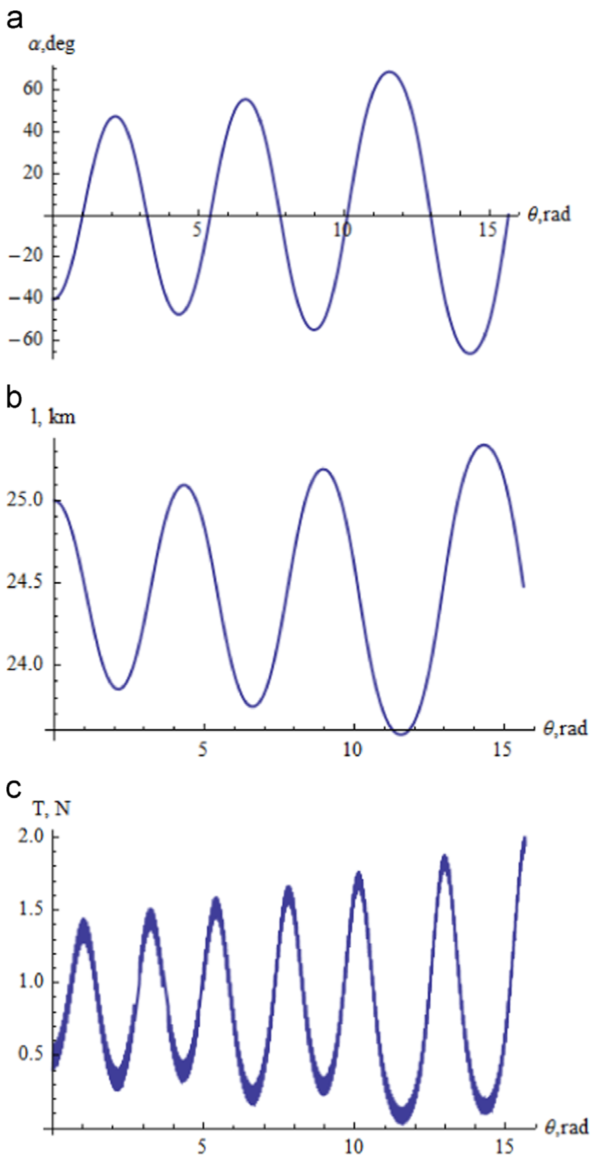

Fig. 2. The dependence of the deflection angle α from true anomaly (a), the dependence of the tether length from true anomaly (b), the dependence of the tether tension from true anomaly (c).

Table 2
The choice of the control coefficient λ.

Control coefficient λ, m	250	500	**750**	1000	1250
Change in altitude Δh, km	−333.0	−334.7	**−329.1**	−323.5	305.4
θ – duration of the deployment of the system θ_k, rad	49.52	26.17	**15.72**	10.67	5.56
Maximum of the tether tension force $T_{\max}$, N	2.01	2.04	**1.97**	1.90	1.69
Maximum rate of release (pull) tether $\dot{l}_{\max}$, m/s	0.49	0.98	**1.42**	1.81	1.99
Variation range of the tether length ($l_{\min}$, $l_{\max}$), km	24.50, 25.15	23.97, 25.30	**23.55, 25.35**	23.16, 25.39	22.96, 25.24

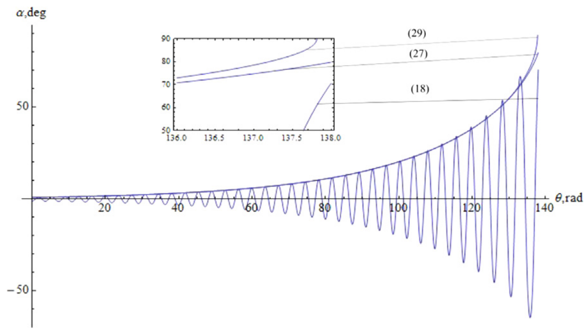

Fig. 3. The dependences of the deflection angle α from true anomaly that are found using.

5. Conclusion

The control law for deployment of a tether-assisted return mission of a re-entry capsule is proposed. The control law is based on the principle of the swing. The approximate analytical solution for the envelope of the deflection angle of the tether from the local vertical is obtained. The numerical simulations show that using of the control law can reduce tether length to approximately 5 km as compared with YES2 mission. The orbital eccentricity and viscoelastic properties of the tether are incorporated into the mathematical model of motion of the tether system. The numerical modeling show that the control law can be effective for the final phase of the tether deployment where the initial deployment was performed by static or dynamic modes.

In general, we believe that the above approach to control of the tether deployment can provide good results for a large variety of applications. Further research on the subject should verify the tether dynamics in more detail.

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References

[1] V.V. Beletsky, E.M. Levin, Dynamics of Space Tether SystemsUnivelt, Univelt Inc, San Diego, CA.
[2] E.M. Levin, Dynamic Analysis of Space Tether MissionsUnivelt Incorporated, San Diego, CA.
[3] V.S. Aslanov, A.S. Ledkov, Dynamics of the Tethered Satellite Systems, Elsevier, Cambridge, MA, 2012.
[4] F. Zimmermann, U.M. Schoettle, E. Messerschmid, Optimization of the tether-assisted return mission of a guided re-entry capsule, Aerosp. Sci. Technol. 9 (2005) 713–721.
[5] L. Johnson, R.D. Estes, E. Lorenzini, M. Martinez-Sanchez, Propulsive small expendable deployer system experiment, J. Spacecr. Rocket. 37 (2) (2000) 173–176J. Sanmartin.
[6] M. Kruijff, Tethers in Space (Ph.D. thesis), Delft University of Technology, The Netherlands, 2011.
[7] P. Williams, A. Hyslop, M. Stelzer, M. Kruijff, YES2 optimal trajectories in presence of eccentricity and aerodynamic drag, Acta Astronaut. 64 (2009) 745–769.
[8] V.S. Aslanov, S.P. Bezglasnyi, J. Appl. Math. Mech. 76 (4) (2012) 405–412.
[9] V.S. Aslanov, S.P. Bezglasnyi, Mech. Solids 47 (3) (2012) 285–297.
[10] A.K. Misra, Dynamics and control of tethered satellite systems, Acta Astronaut. 63 (2008) 1169–1177.
[11] P. Williams, C. Blanksby, P. Trivailo, Tethered planetary capture maneuvers, J. Spacecr. Rocket. 41 (2004) 603–613.
[12] G.A. Corn, T.M. Corn, Mathematical Handbook, McGraw-Hill, New York, 1968.